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Perfect Tracking Control Based on Multirate Feedforward Control and Applications to Motion Control and Power Electronics
– A Simple Solution via Transfer Function Approach –

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Abstract

In this paper, a novel perfect tracking control method based on multirate feedforward control is proposed. The advantages of the proposed method are 1) the proposed multirate feedforward controller eliminates the notorious unstable zero problem in designing the discrete-time inverse system, 2) the plant states track the desired trajectories with zero error at every sampling point of reference input, and 3) the feedback characteristics are completely independent of the proposed controller. Thus, highly robust performance is assured by the robust feedback controller. Moreover, the proposed design method via transfer function approach is much simpler than that of the state space approach which was previously presented by authors. Illustrative examples of position control using servomotor, hard disk drive, and voltage control of an inverter are presented, and simulations and experiments demonstrate the advantages of this approach.

Key words: digital control, multirate sampling, tracking, motion control, power electronics

1 Introduction

In digital control systems, tracking controllers are often employed because the controlled plant follows a smoothed desired trajectory. The best tracking controller is ideally perfect tracking controller (PTC) which controls the object with zero tracking error [1]. Perfect tracking control can be achieved using d-step preview action and a feedforward controller $C_1[z]$ which is realized by an inverse of the closed-loop system $G_{cl}[z]$.

$$C_1[z] = \frac{1}{z^d G_{cl}[z]} = \frac{1 - P[z]C_2[z]}{z^d P[z]} \tag{1}$$

$$r[i] = y_d[i+d] \tag{2}$$

Here, d is the relative degree of $G_{cl}[z]$, $r[i]$ is the reference input, $y_d[i]$ is the desired trajectory, and $C_2[z]$ is the feedback controller.

However, the discrete-time plant $P[z]$ discretized by zero-order hold usually has unstable zeros [2]. Thus, $C_1[z]$ becomes unstable because $G_{cl}[z]$ has unstable zeros. Therefore, in conventional digital control systems utilizing zero-order holds, perfect tracking control is usually impossible.

From this viewpoint, two feedforward control methods have been proposed for the discrete-time plant with unstable zeros [1]. First, the stable pole zero canceling (SPZC) controller cancels all poles and stable zeros of the closed-loop system, which has both phase and gain errors caused by the uncancellable unstable zeros. Second, the zero phase error tracking controller (ZPETC) adds the factors which cancel the phase error, to SPZC. However, the gain error caused by the unstable zeros remains.

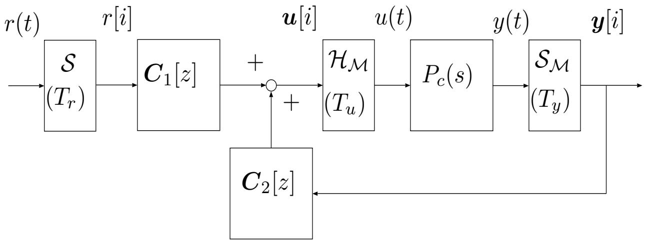


Fig. 1. Two-degree-of-freedom control system.

On the other hand, authors developed perfect tracking control method using multirate feedforward control instead of the zero-order hold [3, 4]. In this paper, a simple design method of perfect tracking controller is presented by transfer function approach. In the perfect tracking control, the tracking error of plant state becomes completely zero at every sampling period of reference input for a nominal plant without disturbance[1]. Moreover, by combining the proposed feedforward controller with a robust feedback controller such as disturbance observer or H_∞ controller, high tracking performance is preserved even if the plant has modeling error and disturbance.

The unstable-zeros problem of the discrete-time plant has been resolved by zero assignment based on multirate control [5, 6]. However, it has been shown that those methods sometimes have the disadvantages of large overshoot and oscillation in the intersample points because the control input changes back and forth very quickly [7]. On the other hand, the proposed method never has this problem because all of the plant states (e.g., position and velocity) are controlled along the smoothed desired trajectories.

2 Perfect Tracking Control

A digital tracking control system usually has two samplers for the reference signal $r(t)$ and the

[1]The word of "perfect tracking control" is originally defined in [1], which means the plant output perfectly tracks the desired trajectory with zero tracking error at every sampling point.

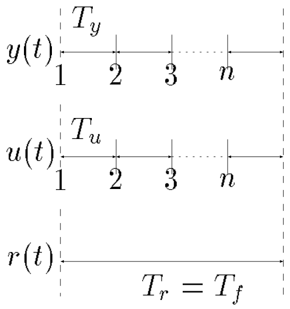


Fig. 2. Multirate feedforward control.

output $y(t)$, and one holder on the input $u(t)$, as shown in Fig. 1. Therefore, there exist three time periods T_r, T_y, and T_u which represent the periods of $r(t), y(t)$, and $u(t)$, respectively [8]. The input period T_u is generally decided by the speed of the actuator, the D/A converter, or the calculations on the CPU. On the other hand, the output period T_y is determined by the speed of the sensor or the A/D converter.

In this paper, the perfect tracking control is proposed in the simplest case for a SISO plant without hardware restrictions on the sampler and holder ($T_y = T_u$) . Because actual control systems usually have restrictions on T_u and/or T_y, the proposed method is extended to general systems with these restrictions ($T_y \neq T_u$) [4].

In the proposed multirate feedforward control, the control input $u(t)$ is changed n times during one sampling period (T_r) of reference input $r(t)$, as shown in Fig. 2. Here n is the plant order. The advantage of the proposed method is that the tracking error of plant state becomes perfectly zero at every T_r.

2.1 Plant Discretization and Parameterization

Consider the continuous-time nth-order plant $P_c(s)$ described by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{b}_c u(t)\ ,\quad y(t) = \boldsymbol{c}_c\boldsymbol{x}(t). \tag{3}$$

The discrete-time plant $P[z_s]$ discretized by the short sampling period T_y $(= T_u)$ of Fig. 3 becomes

$$\begin{aligned} \boldsymbol{x}[k+1] &= \boldsymbol{A}_s\boldsymbol{x}[k] + \boldsymbol{b}_s u[k] &(4)\\ y[k] &= \boldsymbol{c}_s\boldsymbol{x}[k], &(5)\end{aligned}$$

where $\boldsymbol{x}[k] = \boldsymbol{x}(kT_y), z_s := e^{sT_y}$, and

$$\boldsymbol{A}_s := e^{\boldsymbol{A}_c T_y},\quad \boldsymbol{b}_s := \int_0^{T_y} e^{\boldsymbol{A}_c\tau}\boldsymbol{b}_c d\tau,\quad \boldsymbol{c}_s := \boldsymbol{c}_c. \tag{6}$$

Thus, the discrete-time plant $\boldsymbol{P}[z]$ discretized by the multirate sampling control of Fig. 3 can be represented by

$$\begin{aligned} \boldsymbol{x}[i+1] &= \boldsymbol{A}\boldsymbol{x}[i] + \boldsymbol{B}\boldsymbol{u}[i] &(7)\\ \boldsymbol{y}[i] &= \boldsymbol{C}\boldsymbol{x}[i] + \boldsymbol{D}\boldsymbol{u}[i], &(8)\end{aligned}$$

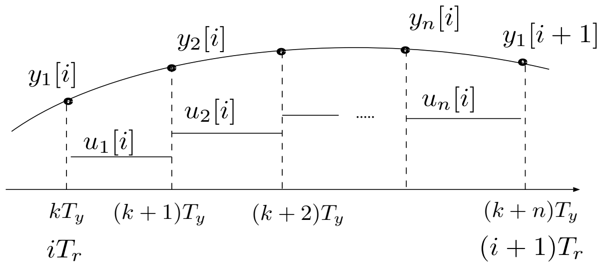


Fig. 3. Multirate sampling control

where $\boldsymbol{x}[i] = \boldsymbol{x}(iT_r), z := e^{sT_r}$, and multirate input and output vectors $\boldsymbol{u}, \boldsymbol{y}$ are defined as [2]

$$\begin{aligned} \boldsymbol{u}[i] &:= [u_1[i], \cdots, u_n[i]]^T\\ &= [u(kT_y), \cdots, u((k+n-1)T_y)]^T, &(9)\\ \boldsymbol{y}[i] &:= [y_1[i], \cdots, y_n[i]]^T\\ &= [y(kT_y), \cdots, y((k+n-1)T_y)]^T, &(10)\end{aligned}$$

and matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ are given by

$$\left[\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{B}\\ \hline \boldsymbol{C} & \boldsymbol{D}\end{array}\right] :=$$

$$\left[\begin{array}{c|ccccc} \boldsymbol{A}_s^n & \boldsymbol{A}_s^{n-1}\boldsymbol{b}_s & \boldsymbol{A}_s^{n-2}\boldsymbol{b}_s & \cdots & \boldsymbol{A}_s\boldsymbol{b}_s & \boldsymbol{b}_s\\ \hline \boldsymbol{c}_s & 0 & 0 & \cdots & 0 & 0\\ \boldsymbol{c}_s\boldsymbol{A}_s & \boldsymbol{c}_s\boldsymbol{b}_s & 0 & \cdots & 0 & 0\\ \vdots & \vdots & & & \vdots & \\ \boldsymbol{c}_s\boldsymbol{A}_s^{n-1} & \boldsymbol{c}_s\boldsymbol{A}_s^{n-2}\boldsymbol{b}_s & \boldsymbol{c}_s\boldsymbol{A}_s^{n-3}\boldsymbol{b}_s & \cdots & \boldsymbol{c}_s\boldsymbol{b}_s & 0\end{array}\right]. \tag{11}$$

The proposed method employs the multirate-input control as a two-degree-of-freedom control, as shown in Fig. 1. In the figures, $\mathcal{H}_\mathcal{M}$ and $\mathcal{S}_\mathcal{M}$ represent the multirate hold and the multirate sampler, respectively. The functions of $\mathcal{H}_\mathcal{M}$ and $\mathcal{S}_\mathcal{M}$ are shown in Fig. 3, and defined in (9) and (10).

In the ideal tracking control system, the transfer characteristic (G_{yr}) from the command r to the output y is generally 1. In this paper, the feedforward controller $\boldsymbol{C}_1[z]$ is considered so that the transfer characteristic from the desired state $\boldsymbol{x}_d$ to the plant state $\boldsymbol{x}$ can be $\boldsymbol{I}$.

2.2 Design of the Feedback Controller $\boldsymbol{C}_2[z]$

Before the perfect tracking controller $\boldsymbol{C}_1[z]$ is designed, the feedback controller $\boldsymbol{C}_2[z]$ must be determined. Here, $\boldsymbol{C}_2[z]$ must be a robust controller which renders the sensitivity function $\boldsymbol{S}[z] = (\boldsymbol{I} - \boldsymbol{P}[z]\boldsymbol{C}_2[z])^{-1}$ sufficiently small at the frequency of the desired trajectory. The reason is that the sensitivity function $\boldsymbol{S}[z]$ represents variation of the command response $\boldsymbol{G}_{yr}[z]$ under the variation of $\boldsymbol{P}[z]$ [10].

For systems without special hardware restrictions in which the feedback loop is single-rate ($T_y = T_u$), the feedback controller $C_2[z_s] = \{\boldsymbol{A}_{s2}, \boldsymbol{b}_{s2}, \boldsymbol{c}_{s2}, d_{s2}\}$ is designed for $P_c(s)$ with a single-rate sampling period T_y $(= T_u)$, where $z_s = e^{sT_y}$. Subsequently,

[2] The operations of (9) and (10) are called "discrete-time lifting" in advanced sampled-data control theory [9] .

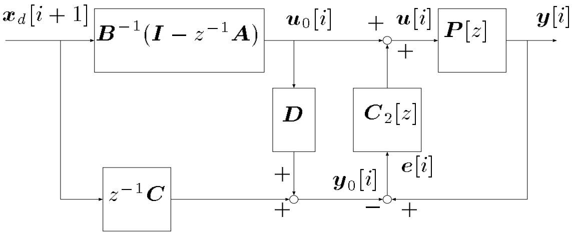


Fig. 4. Perfect tracking controller by the transfer function approach.

$C_2[z_s]$ is transferred to an n-input n-output system $\boldsymbol{C}_2[z]$ using (12), in order to realize $\boldsymbol{C}_1[z]$ and $\boldsymbol{C}_2[z]$ together, where $z = e^{sT_y} = z_s^n$.

$$\boldsymbol{C}_2[z] = \left[\begin{array}{c|ccccc} \boldsymbol{A}_{s2}^n & \boldsymbol{A}_{s2}^{n-1}\boldsymbol{b}_{s2} & \boldsymbol{A}_{s2}^{n-2}\boldsymbol{b}_{s2} & \cdots & \boldsymbol{b}_{s2} \\ \hline \boldsymbol{c}_{s2} & d_{s2} & 0 & \cdots & 0 \\ \boldsymbol{c}_{s2}\boldsymbol{A}_{s2} & \boldsymbol{c}_{s2}\boldsymbol{b}_{s2} & d_{s2} & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ \boldsymbol{c}_{s2}\boldsymbol{A}_{s2}^{n-1} & \boldsymbol{c}_{s2}\boldsymbol{A}_{s2}^{n-2}\boldsymbol{b}_{s2} & \boldsymbol{c}_{s2}\boldsymbol{A}_{s2}^{n-3}\boldsymbol{b}_{s2} & \cdots & d_{s2} \end{array}\right] \tag{12}$$

Because the feedback characteristics such as disturbance rejection performance and stability robustness are never improved by the multirate control in the case where there is no hardware restriction in the sampling scheme ($T_y = T_u$) [11, 12], it is not necessary to design a n-input n-output multirate system as the feedback controller $\boldsymbol{C}_2[z]$. Therefore, a single-rate feedback controller $C_2[z_s]$ is adequate in the case of $T_y = T_u$.

2.3 Design of the Perfect Tracking Controller $\boldsymbol{C}_1[z]$ – Transfer Function Approach

In this section, the perfect tracking controller is designed using the transfer function approach, which can be understood more intuitively than the state space approach presented in [4]. The proposed controller can assure perfect tracking at every sampling point T_r.

From (7) and (8), the transfer function from $\boldsymbol{x}[i+1]$ to $\boldsymbol{u}[i]$ and $\boldsymbol{y}[i]$ is described by

$$\begin{aligned} \boldsymbol{u}[i] &= \boldsymbol{B}^{-1}(\boldsymbol{x}[i+1] - \boldsymbol{A}\boldsymbol{x}[i]) \\ &= \boldsymbol{B}^{-1}(\boldsymbol{I} - z^{-1}\boldsymbol{A})\ \boldsymbol{x}[i+1] \\ &= \left[\begin{array}{c|c} \boldsymbol{O} & -\boldsymbol{A} \\ \hline \boldsymbol{B}^{-1} & \boldsymbol{B}^{-1} \end{array}\right] \boldsymbol{x}[i+1] \quad (13) \\ \boldsymbol{y}[i] &= z^{-1}\boldsymbol{C}\ \boldsymbol{x}[i+1] + \boldsymbol{D}\ \boldsymbol{u}[i]. \quad (14) \end{aligned}$$

In (13), the nonsingularity of matrix $\boldsymbol{B}$ is assured for controllable plant, because $\boldsymbol{B}$ in (11) is equal to the controllability matrix. Because all poles of the transfer function (13) are zero, it is found that (13) is a stable inverse system. Thus, if the control input is calculated by (15) as shown in Fig. 4, perfect tracking is guaranteed because (15) is an exact inverse plant.

$$\boldsymbol{u}_0[i] = \boldsymbol{B}^{-1}(\boldsymbol{I} - z^{-1}\boldsymbol{A})\ \boldsymbol{x}_d[i+1] \tag{15}$$

Here, $\boldsymbol{x}_d[i+1]$ is previewed desired trajectory of plant state. The output of the nominal plant model can be calculated by

$$\boldsymbol{y}_0[i] = z^{-1}\boldsymbol{C}\boldsymbol{x}_d[i+1] + \boldsymbol{D}\boldsymbol{u}_0[i]. \tag{16}$$

When the tracking error $\boldsymbol{e}$ is caused by disturbance or modeling error, it can be eliminated using the robust feedback controller $\boldsymbol{C}_2[z]$ by applying (17).

$$\boldsymbol{u}[i] = \boldsymbol{u}_0[i] + \boldsymbol{C}_2[z](\boldsymbol{y}[i] - \boldsymbol{y}_0[i]) \tag{17}$$

3 Application to motion control

3.1 Tracking control of servomotor

In this section, the proposed perfect tracking control method is applied to the position control system of the servomotor in a two-link direct-drive robot manipulator.

The servomotor with current control is described by

$$P_c(s) = \frac{K}{Js^2}. \tag{18}$$

The feedback controller $C_2[z]$ is obtained from the continuous-time H_∞ mixed-sensitivity problem and Tustin transformation.

Simulated and experimental results are shown in Fig. 5 and Fig. 6. The desired trajectory is a sinusoidal waveform represented by

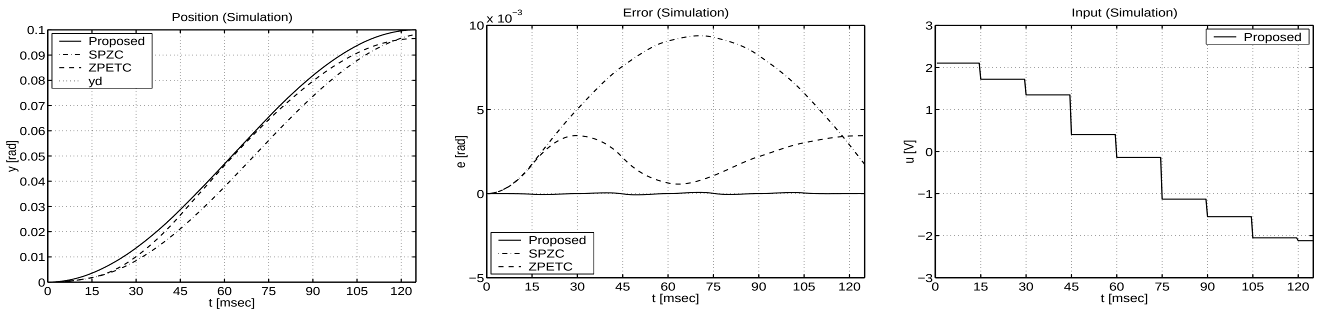
$$\begin{aligned} \theta_d(iT_r) &= A(1 - \cos(\omega_{ref}\ iT_r)) \\ \omega_d(iT_r) &= A\omega_{ref}\sin(\omega_{ref}\ iT_r), \end{aligned} \tag{19}$$

where $\omega_{ref} = 2\pi \times 4$[rad/s]. In this system, both the input and output periods are $T_y = T_u = 15$[ms][3]. Because this plant is a 2nd-order system, the sampling period of the reference signal becomes $T_r = 30$[ms] ($N = 2$).

In the following simulations and experiments, the proposed method is compared with the SPZC and ZPETC proposed in [1], with the same T_y and T_u. The reference sampling period T_r of the proposed method is set twice as long as those of SPZC and ZPETC, because these methods are single-rate approaches and sampling periods are set to $T_y = T_u = T_r = 15$[ms]. However, the proposed controller utilizes the desired trajectories of both position and velocity, while SPZC and ZPETC use those of position only.

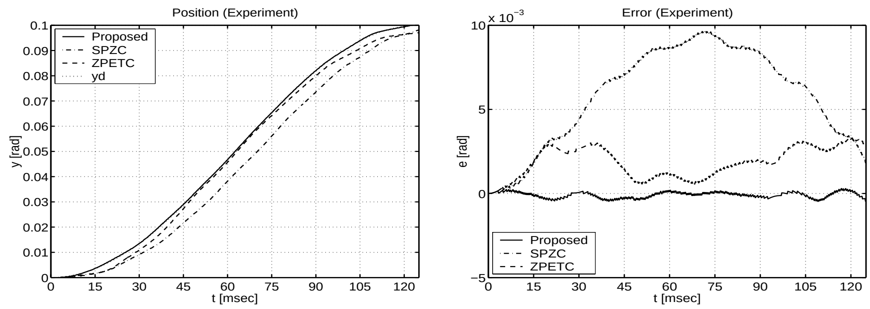
Fig. 5(a) and (b) show that the proposed method exhibits better performance than either SPZC or ZPETC. While the responses of SPZC and ZPETC include large tracking errors caused by the unstable zero, those of the proposed method have zero tracking error. The simulated time response of the control input is shown in Fig. 5(c), which indicates that the control input of the proposed method is smooth despite using multirate input control. Thus, we find that the proposed multirate feedforward method is very practical. Moreover, as shown in Fig. 6, the experimental result also indicates that the proposed method has high tracking performance. Fig. 5 and Fig. 6 also show that the intersample responses are very smooth, because not only position but also velocity follows the desired trajectories at every sampling point T_r.

[3]In the experimental results (Fig. 6), the output signals are sampled at much shorter than 15 [ms] in order to display the intersample responses. The sampling period is set relatively long so as to make the comparison clear.



(a) Position (b) Tracking error (c) Input

Fig. 5. Simulation results ($T_y = T_u = 15$[ms])



(a) Position (b) Tracking error

Fig. 6. Experimental results ($T_y = T_u = 15$[ms])

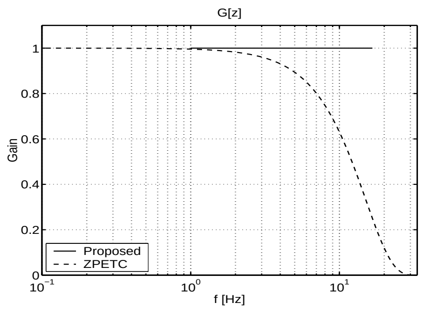


Fig. 7. Frequency response $y[z]/y_d[z]$

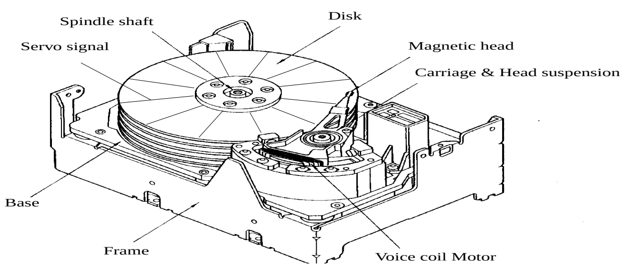
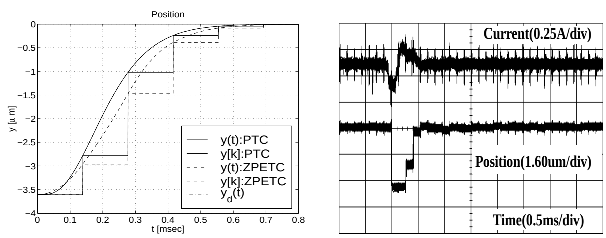


Fig. 8. Hard disk drive.



(a) Simulation (b) Experiment

Fig. 9. Seeking control of HDD

The frequency responses from the desired trajectory $y_d[i]$ to the output $y[i]$ are shown in Fig. 7. Because the proposed method ensures perfect tracking control, the command response becomes 1 for all frequencies. In comparison, the gain of ZPETC decreases at high frequencies.

3.2 Seeking control of hard disk drive

In the servo systems of hard disk drives, the head position is detected by the discrete servo signals embedded in the disks, as shown in Fig. 8. Therefore, the output sampling period T_y is decided by the number of these signals and the rotational frequency of the spindle motor. However, it is possible to set the control period T_u shorter than T_y because of the recent development of CPU. Thus, the controller can be regarded as the multirate system which have the hardware restriction of $T_u < T_y$.

In this section, the proposed PTC is applied to seeking control of 3.5-in hard disk drive [13, 14]. The plant is modeled by double integrator system with time delay. Thus, the proposed method has extended to the hardware restriction of $T_u < T_y$ [4] and time delay [8, 14]. The sampling time of this drive is $T_y = 138.54$ [μs], and the control input can be changed $N = 4$ times during this period. The actual plant has the first mechanical resonance mode around 2.7 [kHz]. The Nyquist frequency is also 3.6 [kHz]. In spite of those, the target seeking-time is set to 3 sampling time (2.4 [kHz]) for one track seeking in these experiments.

Perfect tracking controller is designed on input multiplicity $N = 4$. Because the plant is second order system ($n = 2$), perfect tracking is assured $N/n = 2$ times during sampling period [4].

Fig. 9(a) shows simulation results of the short-span seeking (1 [trk]), which shows that the proposed method gives better performance than ZPETC. While the response of ZPETC has large tracking

Table 1. Experimental seeking-time.

	PTC	ZPETC	Conventional
1trk	$3.17T_s$	$3.77T_s$	$4.14T_s$
6trk	$8.66T_s$	$9.57T_s$	$14.0T_s$

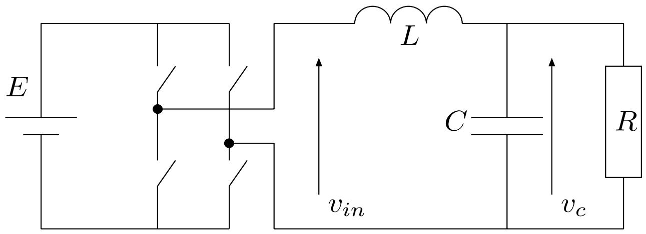


Fig. 10. Inverter system.

error caused by the unstable zero, that of the proposed method has almost zero tracking error. As a result, the proposed method exhibits very high speed seeking. Fig. 9(b) shows that the seeking time of the proposed method gets to about 3 sampling time in the experiments.

Table 1 shows the average seeking-time which is measured in the 2000 times experiments. The seeking-time of the proposed method (PTC) is much smaller than that of ZPETC and the conventional settling control [15]. In the short-span seeking (1 [trk]), the seeking-time of the proposed method is 19 and 31 [%] shorter than the ZPETC and conventional method, respectively. In the middle-span seeking (6 [trk]), the proposed method is 1 and 6 sampling time faster than them. The details of these experiments have been presented in [14].

4 Application to power electronics

In this section, the proposed method is applied to the voltage control of an inverter. Tracking performance is very important not only in motor drive but also in active filter and UPS. The controlled plant is shown in Fig. 10. The PWM inverter bridge can generate output voltage of $\pm E$ or 0, and this system has a LC filter and a resistive road R. This plant is modelled by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{b}_c v_{in}(t)\ ,\quad y(t) = \boldsymbol{c}_c\boldsymbol{x}(t), \tag{20}$$

$$\boldsymbol{A}_c := \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{CR} \end{bmatrix},\quad \boldsymbol{b}_c := \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix},$$
$$\boldsymbol{c}_c := [1, 0],\quad \boldsymbol{x} := \begin{bmatrix} \boldsymbol{v}_c \\ \dot{\boldsymbol{v}}_c \end{bmatrix}, \tag{21}$$

where $R = 2[\Omega], L = 0.53[\text{mH}],\ C = 800[\mu\text{F}], E = 40[\text{V}]$ [16]. Selecting the pulse width $\Delta T[k]$ in Fig. 11 as control input $u[k]$, the discrete time plant with control period $T_u (= T_y)$ is modelled by (4), and the parameters are described by [16, 17]

$$\boldsymbol{A}_s := e^{\boldsymbol{A}_c T_u},\ \boldsymbol{b}_s := e^{\boldsymbol{A}_c T_u/2}\boldsymbol{b}_c E,\ \boldsymbol{c}_s := \boldsymbol{c}_c, \tag{22}$$

where $T_u = 0.1[\text{ms}]$. In the proposed multirate feedforward control, the sampling period of the

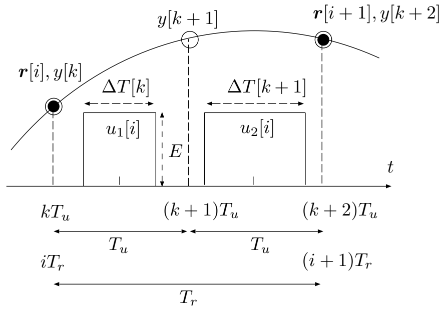


Fig. 11. Multirate PWM control.

reference input is twice as long as the control period ($T_r = 2T_u$), because the plant is a second order system.

The simulation results for sinusoidal desired trajectory are shown in Fig. 12 $\sim$ 17. Fig. 12 indicates that the conventional single-rate deadbeat controller has tracking error even on the sampling points, because it requires two-step settling time. The frequency of desired trajectory is assumed to be unknown. Thus, the deadbeat controller is designed for step-type reference signal.

If the desired trajectory is given at every two sampling time, the deadbeat controller still has problems of large intersample tracking error and oscillated control input, as shown in Fig. 13 $\sim$ 15. In this paper, the conventional deadbeat control with $T_r = 2T_u$ is named as multirate deadbeat control. While the reference input of the multirate deadbeat control is output variable $y_d[i]$, the proposed method (PTC) utilises the desired state variable $\boldsymbol{x}_d[i]$. Thus, perfect tracking of not only the output voltage but also its derivative is guaranteed. Therefore, the intersample tracking error of the proposed method is much smaller than that of the deadbeat controller, and the control input is very smooth.

In Fig. 16 and 17, the robustness for plant variation is examined. For inductance variation and time delay, the responses of conventional feedback deadbeat controller become unstable. However, those of proposed controller are stable and perfect tracking performance is maintained.

Because the deadbeat performance is realized by high-gain feedback controller, the stability robustness is weak in conventional deadbeat control [16, 17]. On the other hand, in the proposed method, perfect tracking performance is realized in feedforward controller. Thus, rich robustness is guaranteed by feedback controller, which is completely independent of the feedforward controller.

5 Conclusion

A novel perfect tracking control method using multirate feedforward control was proposed. The advantage of this method is that the feedforward controller can be designed without considering the unstable zero problem. Moreover, by combining the proposed feedforward controller with a robust feedback controller, high robust tracking performance is obtained. The proposed design method via transfer

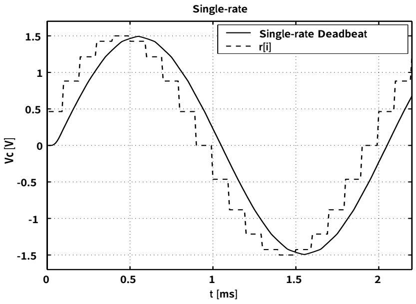


Fig. 12. Conventional single-rate deadbeat control.
($T_r = T_u = 0.1$[ms])

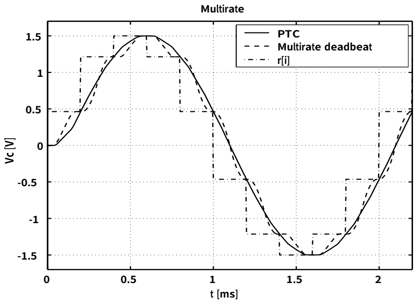


Fig. 13. Proposed method and multirate deadbeat control.
($T_r = 2T_u = 0.2$[ms])

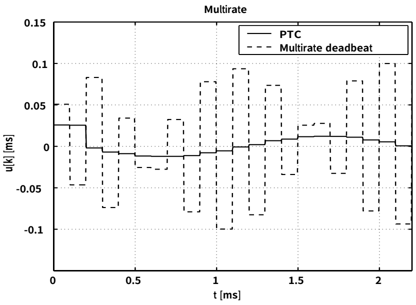


Fig. 14. Control input $u[k] (= \Delta T)$.

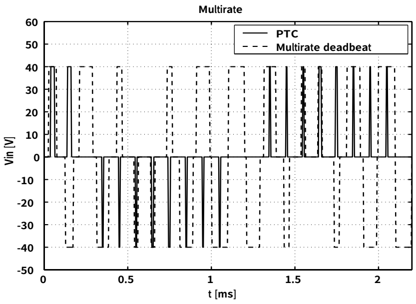


Fig. 15. Output voltage of PWM inverter bridge $v_{in}(t)$.

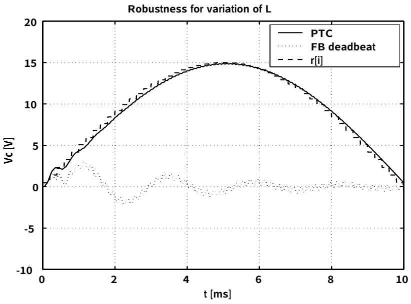


Fig. 16. Robustness for parameter variation ($L = L_n/3$).

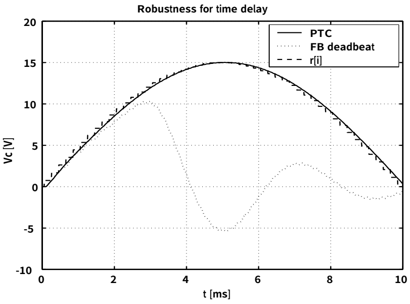


Fig. 17. Robustness for time delay ($T_d = 60$ [μs]).

function approach is much simpler than that of the state space approach which was previously presented by authors. Finally, examples of tracking control of servomotor, seeking control of hard disk drive, and voltage control of an inverter demonstrated the advantages of this approach through simulations and experiments.

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